Instructions: Try these problems from the textbook as practice (answers below).

Problems

11-14. Sketch the solid described by the given inequalities.

11. $\rho \le 1,\ 0 \le \phi \le \pi/6,\ 0 \le \theta \le \pi$
13. $1 \le \rho \le 3,\ 0 \le \phi \le \pi/2,\ \pi \le \theta \le 3\pi/2$

15. A solid inside the sphere $x^2+y^2+z^2 = 4z$ and outside the cone $z = \sqrt{x^2+y^2}$. Write a description of the solid in terms of inequalities involving spherical coordinates.

17. Sketch the solid whose volume is given by $\displaystyle\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho^2 \sin(\phi)\ d\rho\ d\theta\ d\phi$ and evaluate it.

23-36. Use spherical coordinates.

23. Evaluate $\displaystyle\iiint_B (x^2+y^2+z^2)^2\ dV$, where B is the ball with center the origin and radius 5.
25. Evaluate $\displaystyle\iiint_E (x^2+y^2)\ dV$, where E lies between the spheres $x^2+y^2+z^2 = 4$ and $x^2+y^2+z^2 = 9$.
27. Evaluate $\displaystyle\iiint_E xe^{x^2+y^2+z^2}\ dV$, where E is the portion of the unit ball $x^2+y^2+z^2 \le 1$ that lies in the first octant.
29. Find the volume of the part of the ball $\rho \le a$ that lies between the cones $\phi = \pi/6$ and $\phi = \pi/3$.

37. Find the volume of the solid E that lies above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 1$.

43-45. Evaluate the integral by changing to spherical coordinates.

43. $\displaystyle\int_0^1\int_0^{1-x^2}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\ dz\ dy\ dx$
45. $\displaystyle\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\ dz\ dy\ dx$

Answers

15. $\pi/4 \leq \phi \leq \pi/2,\ 0 \leq \rho \leq 4\cos\phi$

23. $\dfrac{312,500\pi}{7}$

25. $\dfrac{1688\pi}{15}$

27. $\dfrac{\pi}{8}$

29. $\dfrac{(\sqrt{3}-1)\pi a^3}{3}$

37. $\dfrac{\pi(2-\sqrt{2})}{3},\ \left(0, 0, \dfrac{3}{8(2-\sqrt{2})}\right)$

43. $\dfrac{4\sqrt{2}-5}{15}$

45. $\dfrac{4096\pi}{21}$